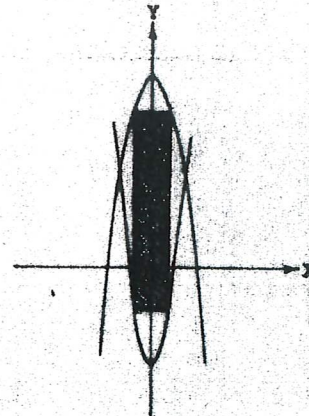


College Entrance Examination Board
 Advanced Placement Examination
 MATHEMATICS: CALCULUS AB
 SECTION II

Time—1 hour and 30 minutes

1. Let $f(x) = \ln(x)$ for all $x > 0$, and let $g(x) = x^2 - 4$ for all real x .
 Let H be the composition of f with g , that is, $H(x) = f(g(x))$.
 Let K be the composition of g with f , that is, $K(x) = g(f(x))$.
- Find the domain of H .
 - Find the range of H .
 - Find the domain of K .
 - Find the range of K .
 - Find $H'(7)$.
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2. Let R be the region in the first quadrant that lies below both of the curves $y = 3x^2$ and $y = \frac{3}{x}$ and to the left of the line $x = k$ where $k > 1$.
- Find the area of R as a function of k .
 - When the area of R is 7, what is the value of k ?
 - If the area of R is increasing at the constant rate of 5 square units per second, at what rate is k increasing when $k = 15$?
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3. Consider $f(x) = \cos^2 x + 2 \cos x$ over one complete period beginning with $x = 0$.
- Find all values of x in this period at which $f(x) = 0$.
 - Find all values of x in this period at which the function has a minimum. Justify your answer.
 - Over what intervals in this period is the curve concave up?
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4. Find the area of the largest rectangle (with sides parallel to the coordinate axes) that can be inscribed in the region enclosed by the graphs of $f(x) = 18 - x^2$ and $g(x) = 2x^2 - 9$.



5. Let R be the region of the first quadrant bounded by the X -axis and the curve $y = 2x - x^2$.
- Find the volume produced when R is revolved around the X -axis.
 - Find the volume produced when R is revolved around the Y -axis.

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6. A particle starts at the point $(5, 0)$ at $t = 0$ and moves along the X-axis in such a way that at time $t > 0$ its velocity $v(t)$ is given by $v(t) = \frac{1}{1+t^2}$.

- (a) Determine the maximum velocity attained by the particle. Justify your answer.
 - (b) Determine the position of the particle at $t = 6$.
 - (c) Find the limiting value of the velocity as t increases without bound.
 - (d) Does the particle ever pass the point $(500, 0)$? Explain.
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7. Let f be the function defined by $f(x) = |x|^{\frac{1}{2}}e^{-x^2}$ for all real numbers x .

- (a) Describe the symmetry of the graph of f .
- (b) Over what intervals of the domain is this function increasing?
- (c) Sketch the graph of f on the axes provided showing clearly:

- (i) behavior near the origin,
 - (ii) maximum and minimum points,
 - (iii) behavior for large $|x|$.
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END OF EXAMINATION